DIMENSIONAL ANALYSIS & ERRORS

Physical quantity

1. The quantities by means of which we describe the laws of physics are called physical quantities. This can also be defined as the quantity that can be measured directly or indirectly.
2. A physical quantity is completely specified if it has:
   (a) Numerical value only i.e. ratio, e.g., dielectric constant, refractive index, etc.
   (b) Magnitude only i.e., scalar, e.g., current, mass, etc.
   (c) Magnitude and direction both i.e., vector e.g. displacement, torque, etc.
3. There are also some such physical quantities, which are not completely specified even by magnitude and direction. Such physical quantities are called tensors, e.g. Moment of Inertia.

Abbreviations for multiples and submultiples

Symbol	Multiplier	Prefix	Symbol	Multiplier	Prefix
D	10^{-1}	deci	da	10^{1}	deca
C	10^{-2}	centi	h	10^{2}	hecto
M	10^{-3}	milli	k	10^{3}	kilo
µ	10^{-6}	micro	M	10^{6}	mega
n	10^{-9}	nano	G	10^{9}	gega
p	10^{-12}	pico	T	10^{12}	tera
f	10^{-15}	femto/fermi	P	10^{15}	peta
a	10^{-18}	atto	E	10^{18}	exa

Fundamental, derived and supplementary units

1. Units of mass, length, time, temperature, electric current, luminous intensity and amount of substance are called basic units.
2. The units of all other physical quantities, which can be derived from fundamental units, are known as derived units.
3. Units of plane angle (Radian) and solid angle (Steradian) are called as supplementary units.

SI system

In this system, there are seven fundamental quantities as shown in table:

Physical quantity	Name	Symbol
Length	metre	M
Mass	kilogram	Kg
Time	second	S
Temperature	kelvin	K
Luminous intensity	candela	cd
Electric current	ampere	A
Amount of substance	mole	mol

Standard of a physical quantity:

1. Standard of length: Most common unit of length is metre. One metre = length of 1650763.73 waves in vacuum of a certain orange-red spectral line of krypton-86.
2. Standard of mass: Most common unit of mass is kilogram. One kilogram = mass of a particular cylinder of platinum-iridium kept in Sevres, France. It is also equal to the mass of 5.0188 x 10^{25} atoms of carbon-12.
3. Standard of time: Most common unit of time is second. One second = duration of 91926331770.0 periods of oscillation of a certain spectral line of cesium-133.

Practical units of length, mass and time

Following table describes some practical units of length, mass and time

Practical unit of length	Practical unit of mass	Practical unit of time
1 light year = 9.46 x 10^{15} m	1 quintal = 10^2 kg	1 year = 365 solar days
1 astronomical unit or l A.U 1.5 x 10^{11} m	1 metric ton = 10^3 kg	1 lunar month = 27.3 solar days
1 parsec = 3.26 light years	1 atomic mass unit (amu) = 1.66 x 10^{-27} kg	1 Solar day = 86400 sec
1 micron = 1 m = 10^{-6} m	1 chandrasekhar limit = 1.4 times the mass of sun = 2.8 x l0^{30} kg	

Some important points concerning dimensional analysis

1. Dimensional constants: Constants having dimensions are known as dimensional constants, e.g., gravitational constant, Planck's constant, universal gas constant, etc.
2. Non-dimensional constants: Constants having no dimensions are known as non-dimensional constants, e.g., mechanical equivalent of heat (J).
3. Dimensional variables: Variable quantities which have dimensions are known as dimensional variables, e.g., velocity, acceleration, force, etc.
4. Non-dimensional variables: Variable quantities having no dimensions are known as non-dimensional variables, e.g., angular displacement, refractive index, etc.
5. Physical quantities having units but no dimensions are angular displacement, plane angle, etc.
6. For given dimension, physical quantity may not be unique, e.g., the dimensional formula [M L^2 T^{-2}] represents work as well as torque.

ERROR ANALYSIS

Significant figures

1. The number of digits in the measured value about the correctness of which we are sure plus one more digit are called significant figures,
2. Rules for counting the significant figures:
   (a) Rule I: All nonzero digits are significant.
   (b) Rule II: All zeros occurring between the nonzero digits are significant. For example, 230089 contains six significant figures.
   (c) Rule III: All zeros to the left of nonzero digits are not significant. For example, 0.0023 contains two significant figures.
   (d) Rule IV: All zeros to the right of nonzero digits are significant. For example, 23.000 as well as 23000 contain five significant figures.
3. In the sum or difference of measurements, we do not retain significant digits in those places after the decimal in which there were no significant digits in any one of the original values.

4. In the product or quotient, we do not retain significant digits more than the least number of significant digits in the values which are multiplied or divided.

Calculation of Errors

1. Precision of measurement: The precision of a measurement depends upon the least count of the measuring instrument. The smaller the least count, the more precise the measurement.
2. Accuracy of measurement: The accuracy of measurement (if there exists an error) depends upon the number of significant figures in it. The larger the number of significant figures, the higher the accuracy. If there is no error in a measurement, then that measurement is most accurate.
   (a) In addition and subtraction, the result cannot be more precise than the least precise measurement.
   (b) In multiplication and division, the result cannot be more accurate than the least accurate measurement.
3. Maximum percentage error:
   (a) For X = A ± B, if ± A, ± B represent errors in A and B, then x, maximum error in x is given by: X = A + B and percentage error is given by $\frac{X}{X} \quad 100 \quad \frac{A \quad B}{A \quad B} \quad 100$
   (b) For $x \quad \frac{a^p b^q}{c^r}$ if percentage errors in a, b and c are small, the maximum percentage error in x is given by: $\frac{x}{x} \; 100 \quad p \frac{a}{a} \quad 100 \quad q \frac{b}{b} \quad 100 \quad r \frac{c}{c} \quad 100$, where percentage errors in a, b and c are taken numerically.

ASSIGNMENT

1. Which of the following is a fundamental quantity
   (a) Volume (b) Velocity
   (c) Time (d) Force
2. Out of the following the only scalar quantity is
   (a) Velocity (b) Force
   (c) Momentum (d) Electric current
3. Which of the following is a derived unit
   (a) Mass (b) Length
   (c) Time (d) Velocity
4. One Poise is equal to
   (a) 0.01 N-s/m^2 (b) 0.1 N-s/m^2
   (c) 1 N-s/m^2 (d) 10 N-s/m^2
5. 3One nanometre is equal to
   (a) 10^9 mm (b) 10^{-6} cm
   (c) 10^{-7} cm (d) 10^{-9} cm
6. A micron is related to centimetre as
   (a) 1 micron = 10^{-8} cm
   (b) 1 micron =10^{-6} cm
   (c) 1 micron = 10^{-5} cm
   (d) 1 micron == 10^{-4} cm
7. The surface tension of a liquid is 70dynes/cm, it may be expressed in MKS system as
   (a) 7 x 10^{-2} newton/metre
   (b) 70 newton/metre
   (c) 7 x 10^2 newton/metre
   (d) 70 x 10^2 newton/metre
8. The SI unit of universal gas constant (R) is
   (a) Watt K^{-1} mol^{-1} (b) N K^{-1} mol^{-1}
   (c) J K^{-1} mol^{-1} (d) erg K^{-1} mol^{-1}
9. Stefan's constant has the unit
   (a) Joule sec^{-1} $metre^{-2}$ $°K^{-6}$
   (b) kg sec^{-3} K^{-4}
   (c) Watt $metre^{-4}$ K^4
   (d) Newton metre sec^{-1} K^{-4}
10. The unit of temperature in S.I. System is
    (a) Degree centigrade
    (b) Degree Celsius
    (c) Kelvin
    (d) Degree Fahrenheit
11. Which one of the following is not measured in units of energy ?
    (a) couple x angle turned through
    (b) moment of inertia x (angular velocity)
    (c) force x distance
    (d) impulse x time
12. Joule x sec is the unit of
    (a) energy (b) momentum
    (c) angular momentum (d) power
13. 3Which of the following can not be expressed as dynes/cm^2 ?
    (a) Pressure
    (b) Longitudinal stress
    (c) Longitudinal strain
    (d) Young's modulus of elasticity
14. The velocity of a particle is given by $v = at^2 + bt + c$ If v is measured in m/s and t is measured in s, the unit of
    (a) a is m/s
    (b) b is m /s
    (c) c is m/s
    (d) a and b is same but that of c is different
15. If the unit of length, mass and time each be doubled, the unit of work is increased
    (a) two times (b) four times
    (c) six times (d) no change
16. The dimensional formula of angular velocity is
    (a) $M^0 L^0 T^{-1}$ (b) $M L T^{-1}$
    (c) $M L T^1$ (d) $M L^0 T^{-2}$
17. The dimensional formula for impulse is
    (a) $M L T^{-2}$ (b) $M L T^{-1}$
    (b) $M L^2 T^{-1}$ (d) $M^2 L T^{-1}$
18. The dimensions of electrical conductivity are
    (a) $M^{-1} L^{-3} T^3 A^2$ (b) $M L^3 T^3 A^2$
    (c) $M^1 L^3 T^{-3} A^{-2}$ (d) $M^2 L^2 T^{-3} A^2$
19. If C and R denote capacity and resistance the dimensions of CR are
    (a) $M^0 L^0 T^1$ (b) $M L^2 T^{-2}$
    (b) $M^0 L^0 T^{-2}$ (d) $M^0 L^0 T^0$
20. The dimension of voltage in form of M L T A are
    (a) $ML^2 T^3 A^{-1}$ (b) $ML^2 T^{-3} A^{-2}$
    (c) $ML^2 T^{-3} A^{-1}$ (d) $ML^2 T^{-3} A^1$
21. Out of the following the only pair that does not have identical dimensions is
    (a) angular momentum and Planck's constant
    (b) moment of inertia and moment of a force
    (c) work and torque
    (d) impulse and momentum
22. The ratios L/R and RC (L= inductance, R = resistance and C = capacitance) have the dimensions as those of
    (a) velocity (b) acceleration
    (c) time (d) force
23. Which of the following is dimensionally correct
    (a) Pressure = Energy per unit area

(b) Pressure = Energy per unit volume
(c) Pressure = Force per unit volume
(d) Pressure = Momentum per unit volume per unit time

24. A student measured the diameter of a wire using a screw gauge with least count 0.001 cm and listed the measurements. The correct measurement is:
(a) 5.320 cm (b) 5.3 cm
(c) 5.32cm (d) 5.3200cm

25. The mass of a body is 20.000 g and its volume is 10.00 cm^3. If the measured values are expressed up to the correct significant figures, the maximum error in the value of density is:
(a) 0.001 g cm^{-3} (b) 0.010 g cm^{-3}
(b) 0.100 gem^{-3} (d) none of these

26. The least count of a stop watch is 0.1 sec. The time of 20 oscillations of the pendulum is found to be 20 sec. The percentage error in the time period is:
(a) 0.25% (b) 0.5% (c) 0.75% (d) 1.0%

27. An experiment measured quantities a, b, c and then x is calculated from $x = ab^2/c^3$. If the percentage errors in a, &, c are ±1%, ±3% and ±2% respectively, the percentage error in x can be:
(a) ±13% (b) ±7% (c) ±4% (d) ±1%

28. The length of a cylinder is measured with a metre rod having least count 0.1 cm. Its diameter is measured with vernier callipers having least count 0.01 cm. Given that length is 5.0 cm and radius is 2.0 cm. The percentage error in the calculated value of the volume will be;
(a) 1% (b) 2% (c) 3% (d) 4%

29. The length, breadth and thickness of a block are measured as 125.5 cm, 5.0 cm and 0.32 cm respectively. Which one of the following measurements is most accurate measurement?
(a) length (b) breadth
(c) thickness (d) height

30. The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimate of the kinetic energy obtained by measuring mass and speed?
(a) 11% (b) 8% (c) 5% (d) 1%

31. While measuring the acceleration due to gravity by a simple pendulum, a student makes a positive error of 1% in the length of the pendulum and a negative error of 3% in the value of time period. His percentage error in the measurement of g by the relation $g = 4\ ^{2}(1/T^{2})$ will be:
(a) 2% (b) 4% (c) 7% (d) 10%

32. The best method to reduce random errors is:
(a) to change the instrument used for measurement
(b) to take help of experienced observer
(c) to repeat the experiment many times and to take the average results
(d) none of the above

33. The random error in the arithmetic mean of 100 observations is x; then random error in the arithmetic mean of 400 observations would be:
(a) 4x (b) x/4 (c) 2x (d) x/2

34. Zero error belongs to the category of:
(a) constant errors (b) personal errors
(c) instrumental errors (d) accidental errors

35. What is the number of significant figures in 0.310 x 10^3?
(a) 2 (b) 3 (c) 4 (d) 6

36. What is the number of significant figures in 200.0?
(a) 1 (b) 2 (c) 3 (d) 4

37. A student performs experiment with simple pendulum and measures time for 10 vibrations. If he measures the time for 100 vibrations, the error in the measurement of time period will be reduced by a factor of:
(a) 10 (b) 90 (c) 100 (d) 1000

38. The length of the rod is measured by different instruments. Which of the following is most accurate result?
(a) 500 mm (b) 500.00 mm
(c) 500.0 mm (d) 0.5 m

39. A body travels uniformly a distance of (13.8 ±0.2) m in a time (4.0 ±0.3) s. The velocity of the body within error limits is:
(a) (3.45 ± 0.2) ms^{-1}
(b) (3.45 ± 0.3) ms^{-1}
(c) (3.45 ± 0.4) ms^{-1}
(d) (3.45 ± 0.5) ms^{-1}

40. The heat dissipated in a resistance can be obtained by the measurement of resistance, the current and time. If the maximum error in the measurement of these quantities is 1%, 2% and 1% respectively, the maximum error in the determination of the dissipated heat is:
(a) 4 % (b) 6 % (c) 4/3 % (d) 2 %

Answers: 1-c, 2-d, 3-d, 4-b, 5-c, 6-d, 7-a, 8-c, 9-a, 10-c, 11-b, 12-c, 13-c, 14-c, 15-a, 16-a, 17-b,18-a, 19-a, 20-c, 21-b, 22-c, 23-b, 24-a, 25-d, 26-b, 27-a, 28-c, 29-c, 30-b, 31-c, 32-c, 33-b, 34-c, 35-b, 36d, 37-a, 38-b, 39-d, 40-b.